A CONING COMPENSATION ALGORITHM WITH PURE FILTERED ANGLE RATE INPUT

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***Abstract*—**In the Strapdown Inertial Navigation System (SINS), the coning error of the inertial measurement unit (IMU) is one of the major factors affecting SINS accuracy, especially for high precision units. To improve the navigation system accuracy with angle rate input, it is necessary to reduce the coning error. This error reduction results from the determination of an exact signal error model, the application of an accurate inertial device, and the use of a high accuracy coning compensation algorithm.

To improve the SINS precision with angle rate input, a coning compensation algorithm with pure filtered angle rate input is developed. It possesses good properties and has none of the drawbacks of competing methods such as the coning compensation algorithm with pure angle rate input and normal 4-order Runge-Kutta algorithm with their requirement for an ideal angle rate input.

According to the different coning effects between the ideal physical quantity and predicted filtered one, a simplified signal error model of the Navigation system is provided and compensation analysis is performed. Combined with the angle increment algorithm using filtered angle rate, compensatory coefficients and the dominant term of the coning residual error compensation algorithm in SINS with the pure filtered angle rate input are deduced. By contrasting simulation results with the characteristics of the navigation system we illustrate that the coning compensation algorithm with filtered angle rate input has better precision by 2 orders of magnitude than traditional attitude algorithm.

The coning compensation algorithm with a pure filtered angle rate input is suited to actual systems, and it provides an important theoretical development and adds project value. The proposed method possesses good properties improving the attitude accuracy of SINS which consists of angle rate Gyros.

***Index Terms*—**attitude algorithm, coning error, rotation vector, error analysis, fiber-optic gyro (FOG)

I. Introduction

During recent years, crucial technology of FOG has been tackled and mastered and lots of results in FOG products have been achieved in China. The SINS prototype, which is based on the China-made FOG, has been processed and developed in many application fields. To make full use of accuracy advantages of FOG, it is necessary to make an early analysis of the algorithm that will be used in FOG SINS.

Under high-speed, high-dynamic circumstances, considering the coupling function, various dithers of the body will cause coning and sculling motion in FOG SINS. Coning motion, equivalent to the net rotation along rotation axis, will result in attitude angle errors, which is the coning error [1] in FOG SINS. Similarly, sculling motion will result in sculling error in SINS. Since the analysis of sculling error can be described in the same way as the coning error [2], further research in coning compensation algorithm with filtered angular rate input is of great significance to improve the attitude accuracy as well as navigation and positioning accuracy of FOG SINS.

Many researchers have conducted lots of research in coning compensation algorithms [3-6], but most of these algorithms relied on the output angle increments of SINS and cannot be directly applied to FOG SINS with angular rate output. Besides, the filtered physical quantity in actual systems is different from the ideal coning motion physical quantity [7-11]. Direct application of ideal angular increments/angular rate coning compensation algorithm with angular rate in project systems cannot improve the attitude accuracy of FOG systems effectively [9].

This paper is aimed to study the FOG SINS with angular rate output, and to propose and derive a coning compensation algorithm based on filtered angular rate input. The simulation analysis of coning motion illustrates the coning compensation algorithm with filtered angular rate input has better performance than other algorithms, and shows that it could be of great reference value to improve the accuracy of SINS.

II. Filtered angular rate coning effect analysis

During data sampling, noise will be mixed in SINS, and hence the output physical quantity of SINS is no longer ideal. Low-pass filter of the noise-mixed FOG data can remove high frequency noise, since effective information is involved in the low-pass signal. Thus the nature of physical quantity is no longer consistent with that of the ideal one [8]. Substituting the filtered angular rate into the coning algorithm with ideal angular increments is unable to realize an excellent coning error compensation effect. So it is necessary to study the derivation of a coning compensation algorithm with a filtered angular rate.

Assuming that the equivalent filter processing function of the output physical quantity from the gyro with a coning angular rate Ω is $F(\Omega)$, which is the digital filter function and has low-pass properties. The principle of the filter function can be expressed as follows:

$$F(0)=1; \quad \lim_{\Omega \to 0} F(\Omega)=1; \quad \left|F(\Omega)\right| \le F_{\max} \tag{1}$$

Defining the function expression $\sin c(x) \triangleq \sin(x)/x$. By combining the FOG sampling period T and the filter function principle mentioned in formula (1), the filter function of actual systems can be approximated by [8]:

$$F(\Omega) \cong \sin c(\Omega T/2) \tag{2}$$

With the effect of the filter function, difference of coning effects between the ideal physical quantity and filtered one is shown in Figure 1.

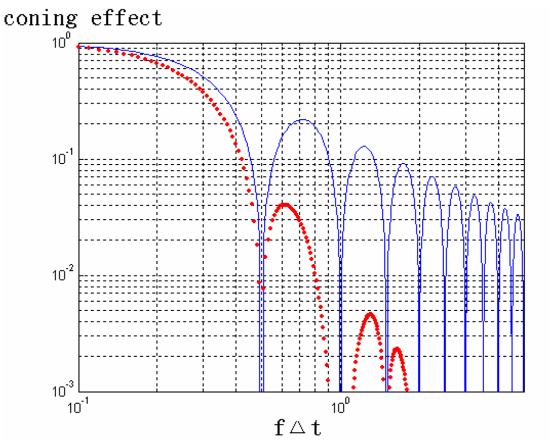

Fig. 1 Difference of coning effect between the ideal physical quantity (continuous line) and project filtered one (dotted line) [8]

In Figure 1, the physical quantity on the abscissa is the product of sampling period and angular frequency, (where $f\Delta t = 1/2$ is the Nyquist frequency point). It can be seen from Figure 1 that, coning effects of the ideal physical quantity (continuous line in Fig 1) and filtered one (dotted line in Fig 1) are different, so the coning compensation algorithm with ideal physical quantity input cannot effectively improve the accuracy of engineering SINS.

III. Filtered physical quantity analysis

Considering modern computers have powerful calculating capacity, one-period coning compensation algorithm is applied in most projects. We discussed the example of one-period M-sample coning algorithm with sampling period T in this paper. To look for the main resource of the coning error, the relationship between filtered angular increment and rotation vector is firstly studied. They are the preparations for the following filtered angular rate algorithm research.

Assuming in coning motion, the motion equation along corresponding axis is $[0, \quad a\cos(\Omega t), \quad a\sin(\Omega t)]^T$, where Ω is angular rate and a is coning half-angle of the coning motion. The gyro angular rate vector [9] in the body coordinate can be written as:

$$\boldsymbol{\omega}_{nb}^{b}(t) = \begin{bmatrix} -2\Omega\sin^2(a/2) \\ -\Omega\sin(a)\sin\Omega t \\ \Omega\sin(a)\cos\Omega t \end{bmatrix} \tag{3}$$

Considering the filter function $F(\Omega)$, the filtered angular rate vector and angular increment vector can be expressed as (4) and (5) respectively.

$$\boldsymbol{\omega}_{nb}^{b}(t) = \begin{bmatrix} -2F(0)\Omega\sin^2(a/2) \\ -\Omega F(\Omega)\sin(a)\sin\Omega t \\ \Omega F(\Omega)\sin(a)\cos\Omega t \end{bmatrix} \tag{4}$$

$$\boldsymbol{\theta}(h) = \Omega h \begin{bmatrix} -2F(0)\sin^2(a/2) \\ -F(\Omega)\sin(a)\sin c(\Omega h/2)\sin[\Omega(t+h/2)] \\ F(\Omega)\sin(a)\sin c(\Omega h/2)\cos[\Omega(t+h/2)] \end{bmatrix} \tag{5}$$

Considering the coning effect, the coning half-angle of the coning motion in the filtered rotation vector can be considered as a function of angular rate Ω, that is $a = a_0 g(\Omega)$ [8], then the rotation vector in the body coordinate can be described as follows:

$$\boldsymbol{\Phi} = \Omega h \begin{bmatrix} -2\sin^2(a_0 g(\Omega)/2)\sin c(\Omega h) \\ -\sin(a_0 g(\Omega))\sin c(\Omega h/2)\sin[\Omega(t+h/2)] \\ \sin(a_0 g(\Omega))\sin c(\Omega h/2)\cos[\Omega(t+h/2)] \end{bmatrix} \tag{6}$$

When Ω and a are small enough [8], the changing relation among rotation vector, angular increment and angular rate satisfy formula (7).

$$\sin^2(a_0 g(\Omega)/2) \approx F^2(\Omega)\sin^2(a/2) \tag{7}$$

Thus rotation vector in the body coordinate can be expressed as

$$\boldsymbol{\Phi} = \Omega h \begin{bmatrix} -2F^2(\Omega)\sin^2(a/2)\sin c(\Omega h) \\ -F(\Omega)\sin(a)\sin c(\Omega h/2)\sin[\Omega(t+h/2)] \\ F(\Omega)\sin(a)\sin c(\Omega h/2)\cos[\Omega(t+h/2)] \end{bmatrix} \tag{8}$$

Comparing equations（5）and（8）, we can see that the components of angular increment and rotation vector along axis Y and Z change periodically, which will not cause the unlimited volatility of attitude angle. The constant component along the X axis directly causes the attitude angle error, so the coning compensation algorithm is specially needed to improve the performance of compensate axis X. The error principle can be defined as

$$\varepsilon = \boldsymbol{\Phi}_x - \hat{\boldsymbol{\theta}}_x - \delta\hat{\boldsymbol{\Phi}}_x \tag{9}$$

Where, $\hat{\boldsymbol{\theta}}$ is the estimated value of angular increment during the period h, and $\delta\hat{\boldsymbol{\Phi}}$ is the estimated value of the rotation vector correction during the period h.

IV. Algorithm for angular increment from filtered angular rate

During the attitude update period from t to $t+h$, gyro angular rate can be expressed [7] by following formula:

$$\boldsymbol{\omega}(t+\tau) = \boldsymbol{g}_0 + \boldsymbol{g}_1\tau + \boldsymbol{g}_2\tau^2 + \cdots + \boldsymbol{g}_M\tau^M \tag{10}$$

Substituting angular rate sample $\boldsymbol{\omega}_i = \boldsymbol{\omega}(t + ih / M)$ into expression (10), we get

$$\boldsymbol{\omega}_i = \boldsymbol{g}_0 + (\frac{i}{M})\boldsymbol{g}_1 h + (\frac{i}{M})^2 \boldsymbol{g}_2 h^2 + \cdots + (\frac{i}{M})^M \boldsymbol{g}_M h^M \quad (i = 0, 1, \cdots M) \tag{11}$$

From the $M+1$ equations in (11), the matrix equation can be obtained as

$$\boldsymbol{W} = \boldsymbol{C}\boldsymbol{G} \tag{12}$$

Where,

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}_0 \\ \boldsymbol{g}_1 h \\ \cdots \\ \boldsymbol{g}_M h^M \end{bmatrix} \text{ and } \boldsymbol{W} = \begin{bmatrix} \boldsymbol{\omega}_0 \\ \boldsymbol{\omega}_1 \\ \cdots \\ \boldsymbol{\omega}_M \end{bmatrix};$$

Thus,

$$\boldsymbol{G} = \boldsymbol{C}^{-1}\boldsymbol{W} \tag{13}$$

From (11), the inner integration angle from t to $t+h$ is,

$$\begin{aligned} \hat{\boldsymbol{\theta}}(h) &= \int_0^h \boldsymbol{\omega}(t+\tau) d\tau \\ &= \boldsymbol{g}_0 h + \frac{1}{2}\boldsymbol{g}_1 h^2 + \frac{1}{3}\boldsymbol{g}_2 h^3 + \\ &\cdots + \frac{1}{M+1}\boldsymbol{g}_M h^{M+1} \\ &= \boldsymbol{H}\boldsymbol{G}h \end{aligned} \tag{14}$$

Substituting （13） into (14), the formula for fitting angular increment is

$$\begin{aligned} \hat{\boldsymbol{\theta}}(h) &= \boldsymbol{H}\boldsymbol{G}h = (\boldsymbol{H}\boldsymbol{C}^{-1})\boldsymbol{W}h \\ &= \boldsymbol{S}\boldsymbol{W}h = \left(s_0\boldsymbol{\omega}_0 + s_1\boldsymbol{\omega}_1 + \cdots + s_M\boldsymbol{\omega}_M\right)h \end{aligned} \tag{15}$$

Where,

$\boldsymbol{S} = \boldsymbol{H}\boldsymbol{C}^{-1} = \begin{bmatrix} s_0 & s_1 & \cdots & s_M \end{bmatrix}$, and $\sum_{i=0}^{N} s_i = 1$ (the details of establishing this expression can be found in reference [12]).

The estimated value of fitting angular increments of axis X can be obtained by substituting (4) into (15), as following

$$\begin{aligned} \hat{\boldsymbol{\theta}}(h)_x &= \left(\boldsymbol{\omega}_{nb}^b(t)\right)_x h \\ &= -2F(0)\Omega \sin^2(a/2)h \\ &= -2\Omega h \sin^2(a/2) \end{aligned} \tag{16}$$

Considering that $h = MT$, and with the help of formula (8) and formula (16), the error along X axis, which is to be corrected, can be described as

$$\boldsymbol{\Phi}_{cx} = 2\Omega MT \sin^2(a/2)\left[1 - F^2(\Omega)\sin c(\Omega MT)\right] \tag{17}$$

Substitute the filter formula (2) and $\lambda = \Omega T$ into formula (17), then describe formula (17) with a Taylor series expansion, formula (18) can be obatined as follows.

$$\begin{aligned} \boldsymbol{\Phi}_{cx} &= 2M\lambda \sin^2(a/2)\left[1 - \sin c^2(\lambda/2)\sin c(M\lambda)\right] \\ &= 2\lambda \sin^2(a/2) \\ &\left\{\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\lambda^{2k}}{(2k+3)!}[(M-1)^{2k+3} - 2M^{2k+3} + (M+1)^{2k+3}]\right\} \end{aligned} \tag{18}$$

V. Coning error coefficients of compensation algorithm with filtered angular rate input

Suppose $\boldsymbol{\omega}_{nM}$ is the M-th sampling value during the n-th attitude update period, p is data interval and T is data sampling interval. The vector cross product $\boldsymbol{C}_p$ will be $\boldsymbol{C}_p = \boldsymbol{\omega}_{nM} \times \boldsymbol{\omega}_{nM+p}$. The X-axis component of $\boldsymbol{C}_p$ could be described as following:

$$\begin{aligned} &\boldsymbol{C}_{px}(n) \\ &= T\omega_{(nM)y} T\omega_{(nM+p)z} - T\omega_{(nM+p)y} T\omega_{(nM)z} \\ &= (\Omega T)^2 F^2(\Omega)\sin^2 a \sin(p\Omega T) \end{aligned} \tag{19}$$

Obviously, formula (19) is only related to p and T.

Substituting the filter formula (2) into formula (19), we obtain:

$$\boldsymbol{C}_{px}(n) = (\Omega T)^2 \sin^2 a \sin c^2(\Omega T/2)\sin(p\Omega T) \tag{20}$$

Set $\lambda = \Omega T$, then

$$\begin{aligned} \boldsymbol{C}_{px}(n) = \sin^2 a\{\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\lambda^{2k+1}}{(2k+1)!}[(p-1)^{2k+1} \\ -2p^{2k+1} + (p+1)^{2k+1}]\} \end{aligned} \tag{21}$$

Suppose the coefficients of the vector cross product are expressed as $x_p(p = 1, 2, \cdots, M)$, then during the time t to $t+h$, the axis X correction of rotation vector can be expressed as:

$$\delta\boldsymbol{\Phi}_x = \sum_{p=1}^{M} \boldsymbol{C}_{px} x_p \tag{22}$$

From formula (18) and formula (22), we obtain:

$$\sum_{p=1}^{M} \boldsymbol{C}_{px} x_p = \boldsymbol{\Phi}_{cx} \tag{23}$$

That is,

$$\begin{aligned} &\sin^2 a \sum_{p=1}^{M}\{\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\lambda^{2k+1}}{(2k+1)!}[(p-1)^{2k+1} - 2p^{2k+1} + (p+1)^{2k+1}]\}x_p \\ &= 2\lambda \sin^2(a/2)\left\{\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\lambda^{2k}}{(2k+3)!}[(M-1)^{2k+3} - 2M^{2k+3} + (M+1)^{2k+3}]\right\} \end{aligned} \tag{24}$$

Given $k = 1, 2, \cdots, M$, the following M equations can be obtained.

$$\begin{aligned} &\sin^2 a \\ &\sum_{p=1}^{M}\{\frac{(-1)^{k+1}\lambda^{2k+1}}{(2k+1)!}[(p-1)^{2k+1} - 2p^{2k+1} + (p+1)^{2k+1}]\}x_p \\ &= 2\lambda \sin^2(a/2) \\ &\left\{\frac{(-1)^{k+1}\lambda^{2k}}{(2k+3)!}[(M-1)^{2k+3} - 2M^{2k+3} + (M+1)^{2k+3}]\right\} \end{aligned} \tag{25}$$

After solving these equations in formula (25), the correction coefficients of compensation algorithm can be obtained as following

$$X_{M\times 1} = [x_1, x_2, \cdots, x_M]^T = A^{-1}B \qquad (26)$$

Where,

$$A_{M\times M} = [(a_{k,p})]_{M\times M} \qquad B_{M\times 1} = [(b_{k,1})]_{M\times 1}$$
$$a_{k,p} = (p-1)^{2k+1} - 2p^{2k+1} + (p+1)^{2k+1}$$
$$b_{k,1} = \frac{(M-1)^{2k+3} - 2M^{2k+3} + (M+1)^{2k+3}}{2(2k+2)(2k+3)}$$
$$k = 1, 2, \cdots, M \qquad p = 1, 2, \cdots, M$$

Because limited vector cross products are used for compensation, a residual error will exist. The dominant term of the residual error can be obtained by subtracting the $M+1$-th order of compensation quantity from ideal value with formula (9) and formula (26). The dominant term of residual error in X axis is as follows:

$$\begin{aligned}\delta\tilde{\phi} &= \boldsymbol{\Phi}_{cx} - \delta\hat{\boldsymbol{\Phi}}_x \\ &= 2\sin^2(a/2)\left\{\frac{(-1)^M \lambda^{2M+3}}{(2M+5)!}[(M-1)^{2M+5} - 2M^{2M+5} + (M+1)^{2M+5}]\right\} \\ &- \sin^2 a\left\{\sum_{p=1}^{M}\{\frac{(-1)^M \lambda^{2M+3}}{(2M+3)!}[(p-1)^{2M+3} - 2p^{2M+3} + (p+1)^{2M+3}]\}x_p\right\}\end{aligned} \qquad (27)$$

Ⅵ. Accuracy analysis and verification of coning algorithm with angular rate input

In a two-sample attitude algorithm with angular rate input, classical "4-order Runge-Kutta quaternion algorithm" is still attractive and widely used [9]. Generally, other two-sample algorithms can improve system precision, but the accuracy improvement is less than one order in magnitude [11]. Herein, coning algorithm precision in this paper is analyzed by comparing with the 4-order Runge-Kutta algorithm.

The dominant term of residual error in 4-order Runge-Kutta algorithm is:

$$-a^2(\Omega h)^5/1440 = -a^2(\Omega T)^5/45 \qquad (28)$$

Comparison between (27) and (28), indicates that the coning compensation algorithm with filtered angular rate input in this paper has better precision than 4-order Runge-Kutta algorithm. To verify this conclusion and make the algorithm analysis clearer, both 2-sample and 4-sample algorithms in this paper and the two-sample 4-order Runge-Kutta algorithm with the same sampling period are contrasted in simulation.

Suppose the coning half-angle is 1 deg and the frequency of the coning motion coupled from body dither is 20Hz, (that is angular rate $\Omega \approx 126\text{rad/s}$). The sampling period of SINS is 1 ms. Under the east-north-up (ENU) geographical coordinate system, the pitch angle of the airplane will generate error drift resulted from coning motion.

Figures 2 and 3 respectively are pitch angular errors of two-sample 4-order Runge-Kutta algorithm and 2-sample filtered algorithm in this paper. It can be seen from the contrast that 2-sample algorithm in this paper will improve the precision by two more orders of magnitude than 4-order Runge-Kutta algorithm, even one more order in magnitude than the improved 2-sample algorithm mentioned in reference [11].

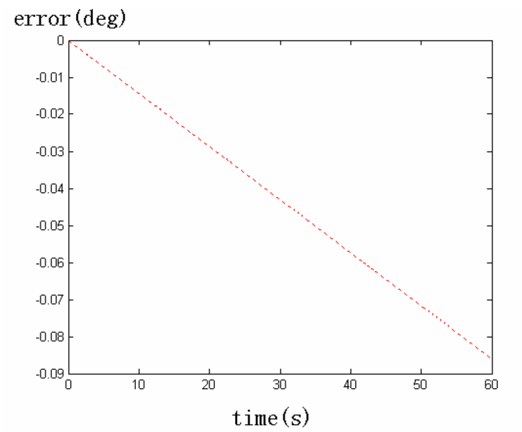

Fig 2. pitch angle error of 4 order Runge-Kutta algorithm

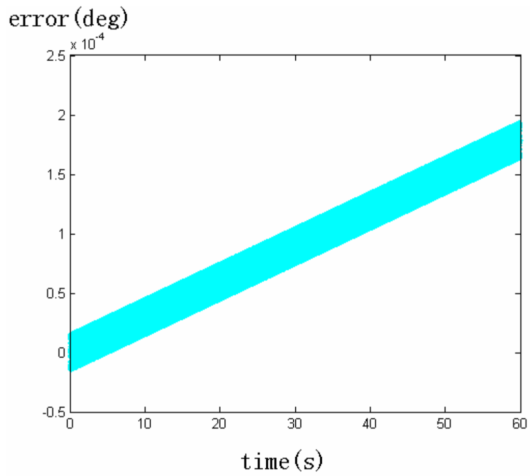

Fig 3. pitch angle error of 2 samples filtered algorithm

Figure 4 shows the pitch angle error of samples filtered coning compensation algorithm with filtered angular rate input. It can be seen that the accuracy is higher than that of the 2-sample algorithm. The high precision 4-sample filtered algorithm in this paper is based on improved compensation coefficients and it shows improved performance.

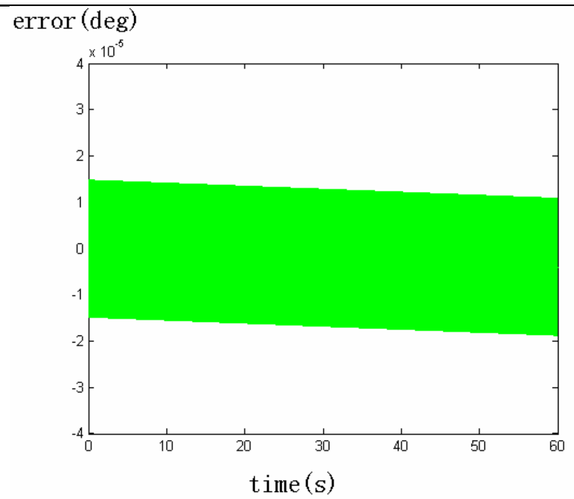

Fig 4. pitch angle error of 4 samples filtered algorithm

Ⅶ. Conclusion

To cooperate with the high accuracy application research of a certain kind of FOG SINS prototype, a coning compensation algorithm with filtered angular rate input is analyzed under engineering circumstances in this paper. The derivation and analysis process of coning error is also described in detail.

The simulation indicates that when the sampling period of the filtered angular rate and the attitude update period are similar, the accuracy of the 2-sample algorithm with filtered angular rate input mentioned in this paper is 2 orders higher in magnitude than the conventional Runge-Kutta algorithm and one order higher in magnitude than common improved two-sample algorithms. The coning compensation algorithm with filtered angular rate input has better performance than other kinds of attitude algorithms, and it is good for improving the accuracy of engineering SINS.

The following research will further analyze 2-sample and 4-sample algorithms with filtered angular rate input, especially in engineering prototype FOG SINS.

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